Name: ____________

Date: ____________

Period: ____________

Student Workbook

Zeros of Polynomial Functions

Learning Target: I can find all real and complex zeros of a polynomial using the Rational Zero Theorem, Complex Conjugate Theorem, and Descartes' Rule of Signs.

Vocabulary

Term	Meaning
Rational Zero Theorem	Possible rational zeros are ±(factors of the constant term)/(factors of the leading coefficient).
Fundamental Theorem of Algebra	A degree-n polynomial has exactly n zeros, counting multiplicity (including complex zeros).
Complex Conjugate Theorem	If a polynomial has real coefficients, complex zeros always come in conjugate pairs (a+bi and a-bi).
Descartes' Rule of Signs	Counts sign changes in f(x) and f(-x) to predict the possible number of positive and negative real zeros.

Key Concept: The Rational Zero Theorem

EXAMPLE 1

List the possible rational zeros of $f(x) = x^3 - 4x^2 - 7x + 10$.

Factors of 10 (constant): ±1, 2, 5, 10   Factors of 1 (leading): ±1

Possible zeros: ____________

Key Concept: Finding All Zeros

METHOD

Test candidates from the Rational Zero Theorem using synthetic division. When you find a zero, the quotient is a lower-degree polynomial — keep factoring until fully solved.

EXAMPLE 2

Find all zeros of $f(x) = x^3 - 4x^2 - 7x + 10$.

Test x=1: $f(1) = 1-4-7+10 =$ ______ → x=1 is a zero.
Dividing gives quotient $x^2-3x-10 = (x-5)(x+2)$. Zeros: ______________

COMMON MISTAKE

Don't forget the ± sign in the Rational Zero Theorem! Every possible zero could be positive OR negative.

Key Concept: Complex Conjugate Zeros

EXAMPLE 3

A degree-3 polynomial with real coefficients has zeros 3 and 2+i. Find the third zero, then write the polynomial in factored form.

Third zero (conjugate): ______
$f(x) = (x-3)(x-(2+i))(x-(2-i))$

Key Concept: Descartes' Rule of Signs

METHOD

Count sign changes in f(x) for the number of POSSIBLE positive real zeros. Count sign changes in f(-x) for the number of POSSIBLE negative real zeros. The actual count is that number, or less by an even amount.

EXAMPLE 4

Apply Descartes' Rule to $f(x) = x^3 - 4x^2 - 7x + 10$ (signs: +, -, -, +).

Sign changes in f(x): ______ → ______ or ______ positive real zeros
$f(-x) = -x^3-4x^2+7x+10$ (signs: -, -, +, +) → sign changes: ______ → ______ negative real zero

Guided Practice — Part A: List Possible Rational Zeros

A1 $f(x) = x^3 - 2x^2 - 5x + 6$

A2 $f(x) = 2x^3 + 3x^2 - 8x - 12$

Guided Practice — Part B: Find All Zeros

B1 $f(x) = x^3 - 2x^2 - 5x + 6$

B2 $f(x) = 2x^3 + 3x^2 - 8x - 12$

Guided Practice — Part C: Complex Conjugate Theorem

C1 A degree-3 polynomial has zeros 2 and 3+i. Find the third zero, then write the polynomial in factored form.

C2 A degree-4 polynomial has zeros -1 (mult. 2) and 4+2i. Find the remaining zero(s) and state the total count of zeros.

Guided Practice — Part D: Descartes' Rule of Signs

D1 $f(x) = x^3 - 2x^2 - 5x + 6$

D2 $f(x) = 2x^3 + 3x^2 - 8x - 12$

BEFORE YOU MOVE ON

Check with a partner: after finding one zero, did you continue dividing the QUOTIENT (not the original polynomial) to find the rest?